A multi-winner election: the 2016 Social Choice and Welfare Council election

Jean-François Laslier and Tobias Rachidi*

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Abstract

This note studies the election of the council of the Social Choice and Welfare society, where 8 seats had to be filled out of 12 candidates, by 63 voters. We compute several voting rules belonging to the class of Generalized Approval Procedures. We also propose a notion of candidate score for such rules, and study the spatial structure of the vote profile.

1 Introduction

For the election of the council of the Social Choice and Welfare society the voting rule is *Simple Approval Voting*: a voter approves as many candidates as she wishes and, with 8 seats to be filled, the 8 candidates with the largest numbers of approvals are elected (ties being broken randomly). But there exist various ways to count approval ballots to elect a fixed number of candidates (Kilgour 2010). In order to understand the differences between these methods, it may be useful to have some real-life examples. This small-size election is an occasion to test various approval-based multi-winner voting rules that have been proposed in the literature.

The 1999 elections at this society have been studied by Brams and Fishburn (2001), Saari (2001), and Laslier (2003). These previous studies were chiefly interested in the amount of strategic voting and in the description of the "Political space" that might be revealed by the correlations of supports among candidates. The conclusions were that strategic voting was not a concern for this election and that the electorate and the candidates looked rather homogeneous, with no clear "political" structure being seen in the vote profile.

In this report, starting from the hypothesis that the approval votes would have been the same if the voting rule were different from Simple Approval Vot-

*Laslier: Paris School of Economics – CNRS, 48, boulevard Jourdan, 75014, Paris, France. E-mail: jean-francois.laslier@ens.fr. Rachidi: Bonn Graduate School of Economics, University of Bonn, Kaiserstr. 1, 53113 Bonn, Germany. Thanks to the ANR projects CoCoRICo-CoDec and ANR-10-LABX-93.

ing,[1] we compute, on the 2016 data, which candidates would have been elected under alternative rules (Section 2). We observe that the set of elected candidates varies with the rule in use. Section 3 introduces a theoretical innovation: we show how to compare the candidates elected under a generalized approval balloting procedure, by decomposing the total welfare associated to the winning committee into marginal contributions at the candidate level. Section 4 analyses the vote profile as was done for the 1999 election. We again see a rather homogeneous electorate but we notice a gender pattern in the spatial structure of the set of candidates.

2 Approval balloting: voting rules and election results

The first part of this section is devoted to the notation and formal definition of multi-winner voting rules based on approval ballots, although we focus on the class of 'Generalized Approval Procedures.' These rules are all extensions of the well known Approval Voting rule from the single-winner case to the multi-winner voting context.

2.1 Definition: generalized approval procedures

There is a set of $m \in \mathbb{N}$ candidates $C = \{1, ..., m\}$ running in the election. The power set of C is denoted by $\mathcal{P}(C)$. There is a finite number $n \in \mathbb{N}$ of voters. Under *approval balloting*, each voter $i \in \{1, ..., n\}$ approves as many candidates he or she wants; the corresponding ballot $V_i \subseteq C$ contains the candidates that voter i approves. The vector $V = (V_1, ..., V_n)$ is called the *ballot profile*. In particular, a voter might approve none of the candidates, or all of them. The set of all possible ballot profiles can be described as $\mathcal{V} = (\mathcal{P}(C))^n$.

For $k \in \mathbb{N}, 1 \leq k \leq m$ we denote by

$$\mathcal{A}_k = \{S \in \mathcal{P}(C) : |S| = k\}$$

the set of committees of size k. A multi-winner voting procedure based on approval ballots is a, possibly multi-valued, function that returns, for every possible ballot profile, one or several winning committees that are elements of the set $\mathcal{A}_k$. Hence, given a ballot profile, such a voting rule selects one or more winning committees with a fixed number, k, of members.[2]

Generalized Approval Procedures are based on a scoring approach to elections. We assign to each feasible committee $S \in \mathcal{A}_k$, a score in the following way. First, we introduce a non-decreasing real-valued sequence r that has as

[1] On strategic voting under approval balloting see Cox (1984), Laslier and Van der Straeten (2016), and Van der Straeten, Lachat and Laslier (2016).

[2] Investigations on the subject include Thiele (1895), Fishburn (1981), Bock, Day and McMorris (1998), Fishburn and Pelec (2004), Brams, Kilgour and Sanver (2007), Laffond and Lainé (2010), Kilgour and Marshall (2012), Elkind et al. (2014) , Faliszewski et al. (2016) , Elkind et al. (2017) .

its first value $r(0) = 0$, and returns for every natural number x from the set $\{1, ..., m\}$ some non-negative real number $r(x)$. Then, the score of a feasible committee is defined as

$$w(S) = \sum_{i=1}^{n} U_i(S) = \sum_{i=1}^{n} r(|V_i \cap S|). \tag{1}$$

Hence, depending on the concrete specification of the sequence r, we can generate several voting rules that are all part of the class of Generalized Approval Procedures. One can think of this score as a *total welfare* that a society derives from a given committee. By definition, the voting rule selects one or more committees that induce the highest score among all feasible committees.

Given a feasible committee, Equation 1 can interpreted as follows. In a first step, we compute for every voter an individual specific score or, in other words, the utility a single voter derives from a committee, although a voter's utility level depends only on the number of candidates in the given committee that this voter approves. Further, as the sequence r is assumed to be non-decreasing, it follows that this individual utility function is non-decreasing in the number of approved alternatives in a given committee. In a second step, we sum these individual values to obtain the score of the committee. Notice that the stated definition does not, in general, allow to proceed directly by computing some candidate specific scores and choosing the k candidates with the highest scores (see below for details).

What are possible configurations of the sequence r? In Table 1 we summarize some specifications proposed in the approval balloting literature. Later, we will apply these voting rules to the council election.

Voting Rule	**Sequence** r, $x \in \{1, ..., m\}$, $r(0) = 0$
Simple Approval	$r(x) = x$
Proportional Approval	$r(x) = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{x} = \sum_{j=1}^{x} \frac{1}{j}$
Majority Threshold	$r(x) = 0,\ x < \frac{k}{2}$ $r(x) = 1,\ x \geq \frac{k}{2}$
Strict Majority Threshold	$r(x) = 0,\ x < \frac{k+1}{2}$ $r(x) = 1,\ x \geq \frac{k+1}{2}$
AV with Majority Threshold	$r(x) = x,\ x < \frac{k}{2}$ $r(x) = \lceil \frac{k}{2} \rceil,\ x \geq \frac{k}{2}$
AV with Strict Majority Threshold	$r(x) = x,\ x < \frac{k+1}{2}$ $r(x) = \lceil \frac{k+1}{2} \rceil,\ x \geq \frac{k+1}{2}$

Table 1: Some 'Generalized Approval Procedures' and their associated sequences

The *Majority Threshold* and *Strict Majority Threshold* rules rest, essentially, on the idea that a voter's utility is zero if he or she does not approve a majority of the candidates in a given committee. When there is such a majority of approved alternatives in the elected committee, the voter's utility becomes one. Particularly, in both situations, a voter's utility is independent of the precise number of approved candidates in a given committee. The procedures *AV with Majority Threshold* and *AV with Strict Majority Threshold* can be seen as a relaxation of the assumption characterizing the previously introduced mechanisms. Hence, here, the utility of a voter increases linearly as under simple Approval Voting, up to some sort of majority threshold. If this threshold is reached, the marginal utility of an additional approved candidate becomes zero. Of course, these specifications should not be seen as complete, but they should rather be considered as an illustration of the wide range of possible configurations.

Proportional Approval Voting (PAV) rests on the following idea. Given a committee, from a voter's perspective, the marginal contribution or utility of an additional alternative in this committee that this voter approves is positive, but it decreases when the number of candidates in this committee approved by the respective voter increases. To capture this general behavior, the specified sequence r is not the only way to model this characteristic, and one might use any increasing sequence of positive numbers with decreasing marginal contributions. Therefore, this voting mechanism somehow reflects the idea of a decreasing marginal utility that is widely used in other contexts in economics. Notice that the Simple Approval Voting rule corresponds, in this framework, to the limiting case in which the mentioned marginal contributions are constant and all equal to 1.

How should the sequence r be specified? Theoretical support has been provided in favor of PAV. As the name recalls, this rule yields proportional representation in the special but theoretically important case of *apportionment*, where disjoint sets of voters approve disjoint sets of candidates (see Brill, Laslier and Skowron (2016)).

Aziz et al. (2017) show that the multi-winner voting rule induced by the specified sequence r based on the harmonic progression is the only configuration among all non decreasing sequences of positive numbers that exhibit non-increasing marginal contributions which guarantees that the corresponding multi-winner voting procedure satisfies their axiom of 'Extended justified representation'. This axiom ensures, given any ballot profile V and any desired committee size k, and for every $l \in \mathbb{N}, 1 \leq l \leq k$, the following property. Suppose that there are at least $l \cdot \frac{n}{k}$ voters such that all these voters simultaneously approve at least l similar alternatives. Then, there has to be at least one voter among the considered voters who has at least l approved candidates in the winning committee. In particular, when adopting this argumentation, Simple Approval Voting, AV with Majority Threshold and AV with Strict Majority Threshold must all be seen as inferior to PAV. However, we cannot infer from their axiomatic work the relationship between the Majority Threshold or Strict Majority Threshold and PAV, because these two rules are based on sequences r that do not exhibit non-increasing marginal utilities. Moreover, given an arbitrary desired committee size k, Simple Approval Voting does not even meet, in general, the weaker condition of 'Justified representation'. This axiom is similar to 'Extended justified representation', although, here, it is only required that the property stated above holds for $l = 1$. Sanchez-Fernandez et al. (2016) criticize the axiom of 'Extended justified representation' and propose an alternative condition that they call 'Proportional justified representation,' but the conclusion regarding the superiority of PAV remains the same.

These results suggest to investigate the effects of PAV and other voting rules distinct from Simple Approval Voting.

2.2 Electoral results for the Social Choice and Welfare council

In this part, we provide the hypothetical election results for the Social Choice and Welfare council when adopting different approval balloting rules. Before that, we briefly describe the setting and the data.

We have $n = 63$ voters, $m = 12$ candidates, and we want to select a committee of exactly $k = 8$ members. Every voter approved at least 1 alternative; 12 voters approved exactly 8 alternatives, meaning these voters approved as many candidates as the voting rule is supposed to select; and 6 voters approved all the 12 candidates running in the election. On average, the 63 voters approved around 6.4 candidates each.

On the basis of the described ballots, we computed for every feasible committee (there are $\binom{12}{8} = 495$ possible combinations) the scores, according to the formula and the specifications of the required sequence, as introduced in the previous section. We provide in Table 2 the winning committees under our six rules. The winning committees are not always unique.

Voting Rule	**Winning Committee(s)**
Simple Approval	A, B, C, D, E, F, G, H
Proportional Approval	A, B, C, D, E, F, G, I
Majority Threshold	A, B, C, D, E, F, G, L A, B, C, D, E, F, G, J
Strict Majority Threshold	A, B, C, D, E, F, H, I
AV with Majority Threshold	A, B, C, D, E, F, G, I
AV with Strict Majority Threshold	A, B, C, D, E, F, G, H A, B, C, D, E, F, G, I

Table 2: Winning committee(s) under different voting rules

Six candidates (A, B, C, D, E and F) would become a member of the council in all cases. Apart from the Strict Majority Threshold rule, such is also the case for G. In addition to that, no voting rule selects K. Comparing PAV and Simple Approval Voting, the table shows that I instead of H wins under PAV. Switching to the Majority Threshold, one can see that the position of I or H goes here to J or L, although the other seven listed candidates would become a member of the council in all three cases. AV with Majority Threshold yields in our application the same outcome as the PAV, and AV with Strict Majority Threshold produces a tie between the unique outcomes of the PAV and of Simple Approval Voting.

To get a more detailed picture of these election results, it may be interesting to compute some scores at the candidate level, in order to somehow quantify the popular support of the candidates one by one. When considering committee scoring functions that are additively separable between the candidates, we can compute such scores for each candidate. In particular, in case of the Simple Approval Voting rule, the score of a candidate equals the number of voters who approve this candidate. However, for instance, regarding PAV, such an approach is not possible because the score of a committee can only be calculated in a holistic way at the committee level.

To tackle this issue, we suggest in the following section a method to decompose the total utility a society derives from the winning committee into contributions of the candidates that are part of this winning committee. The alternatives that are not winning have contributions of zero to the social utility. This procedure might give us a more detailed picture of the magnitudes of what each single candidate brings, by himself to the society. This information is a marginal value, given the presence of the others.

3 Candidates' contributions to the social welfare: method and application

In this section we introduce a new method to decompose the score or welfare that a committee induces into contributions of the candidates that are part of this committee. As stated in the previous section, this approach might be particularly useful when considering multi-winner voting rules that are based on a committee score that is not additively separable between the alternatives. We define this procedure in the context of 'Generalized Approval Procedures'. However, the conceptual idea might also be applicable to multi-winner voting rules that do not rely on approval ballots. In the second part of this section we illustrate the method and provide the decomposition of the social welfare generated by the theoretical winning council of the PAV.

3.1 Method: decomposition of the social welfare

In the case of Simple Approval Voting, where the committee score is additively separable, one would intuitively define the contribution of a candidate to the score of any committee as the number of voters that approved this candidate. In that case the marginal contribution of a candidate is the same in any committee.

To put it another way, given a fixed committee, for each voter, we take the utility that he or she derives from this committee, and divide these individual utilities by the number of candidates in the considered committee that were approved by the voter. Hence, these ratios describe the average utility that a voter derives from her "representatives" in the committee. Now, for each candidate in the committee, we can sum up these average utilities over the voters. For each elected candidate, we only take into account her value for the voters who approved this candidate. This way of thinking leads, in the case of

Simple Approval Voting, exactly to the same result as the direct computation of the scores of the candidates in the fixed committee. This approach implies that the contributions of the candidates outside the considered committee are zero.

We will use this idea to derive the contribution of a candidate to the social welfare in a given committee by computing the average utilities, and we extend it to an arbitrary committee scoring function corresponding to the class of Generalized Approval Procedures. Even if we are mainly interested in the decomposition of the social welfare induced by the winning committee of an approval-based multi-winner voting rule, the following formal definition can be stated for any feasible committee, given a sequence r.

For a formal definition, consider a feasible committee $S \in \mathcal{A}_k$. For any candidate $a \in C$ running in the election, we denote by $w(a, S)$ the part of the social welfare, or score, generated by the feasible committee S that can be attributed to a:

$$w(a,S) = \sum_{i \in \{1,\ldots,n\}: a \in V_i \cap S} \frac{U_i(S)}{|V_i \cap S|}.$$

Of course, if $a \notin S$, there is nothing to sum up and $w(a, S) = 0$.

It can be shown that the sum of the contributions of all candidates, given the feasible committee S, just equals the score or social utility generated by S. In other words: for any sequence r, $\sum_{a \in C} w(a, S) = w(S)$. The subsequent equations are the proof of this proposition; we only use standard equivalent transformations and the fact that $r(0) = 0$.

$$\begin{aligned}
w(S) &= \sum_{i=1}^{n} r(|V_i \cap S|) = \sum_{i:|V_i \cap S| \neq 0} \frac{r(|V_i \cap S|)}{|V_i \cap S|} \cdot |V_i \cap S| \\
&= \sum_{i:|V_i \cap S| \neq 0} \sum_{a \in C} \frac{r(|V_i \cap S|)}{|V_i \cap S|} \cdot \mathbf{1}_{[a \in V_i \cap S]} \\
&= \sum_{a \in C} \sum_{i:|V_i \cap S| \neq 0, a \in V_i \cap S} \frac{r(|V_i \cap S|)}{|V_i \cap S|} \\
&= \sum_{a \in C} \sum_{i: a \in V_i \cap S} \frac{U_i(S)}{|V_i \cap S|} \\
&= \sum_{a \in C} w(a, S).
\end{aligned}$$

It is important to emphasize that, in general, the contribution of a candidate to the score of a committee changes from one committee to another. Thus, we cannot compare contributions across different committees. In particular, we cannot use these contributions to compose the winning committee. It is only possible to compare the different contributions with respect to a given committee. However, in the special case of Simple Approval Voting or, more generally, if we have 'Generalized Approval Procedures' with committee scoring functions that are additively separable between the candidates, this dependency disappears in the following sense. If a candidate is outside a given committee, her contribution is zero. If a candidate is part of a given committee, the magnitude of this candidate's contribution is the same for every committee that contains this candidate. This can be seen from the following observations.

Kilgour (2010) states that a committee scoring function induced by an increasing sequence r is additively separable between the candidates if and only if the respective sequence r satisfies $r(0) = 0$ and, for every $x \in \{1, ..., m\}$, $r(x) = A \cdot x$ for some positive number A, that one can then fix to 1; that is Simple Approval. This observation carries over to the case of the Generalized Approval Procedures that are derived from progressions that are non-decreasing while assuming the number A to be non-negative. Using this result and plugging it into the equation for a contribution of an alternative $a \in C$, given the feasible committee S, yields the desired result:

$$w(a,S) = \sum_{i:a \in V_i \cap S} \frac{r(|V_i \cap S|)}{|V_i \cap S|} = \sum_{i:a \in V_i \cap S} \frac{|V_i \cap S|}{|V_i \cap S|} =$$
$$= \sum_{i:a \in V_i \cap S} 1 = \begin{cases} \sum_{i \in \{1,\dots,n\}:a \in V_i} 1, & a \in S \\ 0, & a \notin S \end{cases} = \begin{cases} w(a), & a \in S \\ 0, & a \notin S \end{cases}$$

Intuitively, the specification $r(x) = x$, $x \in \{1, ..., m\}$ implies that the average utility a voter derives from a candidate that is part of a given committee is constant. Therefore, these average utilities do not depend on the presence of other candidates in the given committee and the contribution of a candidate to the social welfare is the same for every committee containing this alternative.

In the following part of this section we apply the introduced method to the winning council under PAV to get a more detailed picture of the theoretical Social Choice and Welfare councils provided in the previous section.

3.2 Application: council vote under proportional approval balloting

We apply the above method and decompose the social welfare induced by the winning council of PAV. As a complement, we also provide the contributions regarding the winning committee derived from Simple Approval Voting, that is, for the winning alternatives, their direct approval scores. The findings are summarized in Table 3.

The results show that the seven candidates that would become members of the SCW council in both voting situations yield for both voting rules higher contributions than the corresponding eighth candidate that wins only in one situation. Interestingly, when comparing the contributions of the stated seven

Candidate	Approval Voting	Proportional Approval
A	51	24.02
B	46	20.73
C	41	18.50
D	38	17.40
E	35	15.99
F	33	14.83
G	32	14.47
H	29	0
I	0	12.42
J	0	0
K	0	0
L	0	0

Table 3: Candidates' contributions to the social utility at the respective optima

alternatives, it turns out that, in our example, the ranking in terms of contributions coincides for both approval-based voting mechanisms.[3]

In the case of Approval Voting, we know that I would generate a lower contribution than H does when replacing H by I, although the contributions of the other seven winning candidates remain constant. The approval scores of the losing candidates are 28 (I), 27 (J), 20 (L) and 23 (K). However, for the Proportional Approval Balloting, as explained in the previous part of this section, we cannot infer such a conclusion, meaning that the contribution of H is smaller than the contribution of I because the impact of this replacement on the contributions of the other seven winning alternatives remains here ambiguous.

4 Spatial representation of candidates and voters: method and results

In this section we represent visually the voters and the candidates of the Social Choice and Welfare council election in an Euclidean political space. Using approval ballots data, this approach allows to investigate the structure of the set of candidates and the set of voters, which seems to be an important election characteristic to be shared with the public. The first part is dedicated to a short review of the previous work on this question and the used statistical techniques. In the second part we summarize our findings related to the 2016 SCW election.

[3] There is also an obvious correlation with alphabetical order, that we cannot study here.

4.1 Statistical methods for dimensionality reduction based on approval ballots data

Following Laslier (2003), with approval ballots data, one can represent voters and candidates as points in an Euclidean space in the following way. We denote by $a_i^c \in \{0,1\}$ the evaluation of alternative $c \in C$ by voter $i \in \{1,...,n\}$, that is $a_i^c = 0$ if voter i rejects candidate c and $a_i^c = 1$ whenever voter i approves alternative c.

The m candidates lie in the space $\mathbb{R}^n$: for candidate $c \in C$, the associated n-dimensional tuple is:

$$a^c = (a_1^c, ..., a_n^c) \in \mathbb{R}^n.$$

This means that coordinate $i, i \in \{1,...,n\}$ equals to 0 if voter i rejects candidate c and the coordinate is set to 1 in case that voter i approves alternative c. Each axis corresponds to a voter.

Analogously, each voter i is represented by a point in $\mathbb{R}^m$:

$$a_i = (a_i^1, ..., a_i^m) \in \mathbb{R}^m.$$

Each axis corresponds to a candidate.

Based on this high-dimensional spatial representation, we apply statistical methods for dimensionality reduction, namely multidimensional scaling, to identify patterns in the structure of the candidates and voters. These statistical techniques take the distances between the points associated to voters or candidates as the input and they yield a low-dimensional representation of these points such that the distances between the generated points in the low-dimensional space approximately replicate the distances in the high-dimensional space. The precise distance metric has to be specified. To put it another way, multidimensional scaling or the related method of principal component analysis aim at preserving the variance in the data while reducing the dimensions to be able to identify possibly existing subgroups or other patterns.

There are various ways to quantify the distances between candidates, and between voters, from approval data[4] but, for our purposes, namely the analysis of the SCW council election, we will follow Laslier (2003) who investigates the same type of election held in the year 1999 and adopt a simple and intuitive distance measure. We quantify the dissimilarity between two candidates as the number of voters that disagree in their evaluations of the two considered candidates. This allows us to measure the range between two voters in an

[4]Laslier (2006) derives in a probabilistic spatial voting setting the distances between two arbitrary candidates assuming that the utility a voter derives from a candidate is composed of a spatial or purely policy driven ingredient as well as a candidate-specific valence component. Further, he assumes that the density function of the voters' position in the political space is very flat or approximately uniform. Then, he derives an expression for the distance between two arbitrary candidates that depends only on a parameter to be calibrated and, in particular, it depends logarithmically on the approval scores of the candidates to be compared and the number of voters that simultaneously approve the two considered alternatives. These latter three numbers are observed for every pair of alternatives. For applications, see Laslier (2010) or Alós-Ferrer and Granic (2015).

analogous way, that is we compute the number of candidates for which the two voters' evaluations do not coincide.

Formally, the verbally described distances between two candidate points $s \in \mathbb{R}^n$ and $t \in \mathbb{R}^n$ as well as the gap between two voter points $i \in \mathbb{R}^m$ and $j \in \mathbb{R}^m$ in the respective Euclidean space can be captured by $d(s,t) = \sum_{i \in \{1,\ldots,n\}} |a_i^s - a_i^t|$ and $d(i,j) = \sum_{c \in C} |a_i^c - a_j^c|$ respectively. Of course, this metric is just the Minkowski distance with parameter $p = 1$. Moreover, and this is a specific feature of approval ballots, as the coordinates of the vectors associated to voters or candidates only have value 0 or 1, the adopted Manhattan metric coincides here with the square of the Euclidean distance.

Using the defined distance measure, we apply multidimensional scaling to the SCW council election and present in the following a spatial representation of the corresponding candidates and voters.

4.2 Findings: spatial description of the 'Social Choice and Welfare' society in 2016

We apply the approach presented in the previous passage to the approval ballots data of the 2016 'Social Choice and Welfare' council election. Findings are compared with the results obtained with the same techniques on the election held in 1999.

Beginning with the voters, the optimal picture preserving two-dimensional representation of the 63 voters in the political space is displayed in Figure 1. The first principal component accounts for 17.94% of the variance in the data, the first two dimensions capture in total 29.11% of the variance (see the Table in Figure 5 for details). It is not possible to infer any clear pattern, and the electorate seems to be rather homogeneous. Incorporating a third dimension or altering the distance metric does not change this conclusion, and the conclusion was the same for the 1999 election.

Now, we turn to the 12 candidates. Figure 2 shows the first two dimensions of the multidimensional scaling. Additionally, we incorporate a third dimension, the corresponding results are provided in Figures 3 and 4. The first principal component of these optimal spatial representations accounts for 28.91% of the variance in the data, the representation in a two-dimensional plain captures 53.92% of the variance and the first three axis realize together 68.61% (see the Table in Figure 6 for details). Like in the voters' case, the overall characteristics of the candidates' structure does not change when altering the distance measure.

First of all, we can infer from figure 2 and from the shares of explained variance that the candidates' space is not one-dimensional, but we need more dimensions to capture an appropriate share of the variation in the data. Moreover, we can see that the first principal component approximately replicates the approval scores of the candidates. This finding coincides with the previous observations and is easily understood. If the approvals are gathered by the candidates in a symmetric manner from the voters, then what distinguishes two candidates (two points a^c and a^d in $\mathbb{R}^n$) is only their scores.

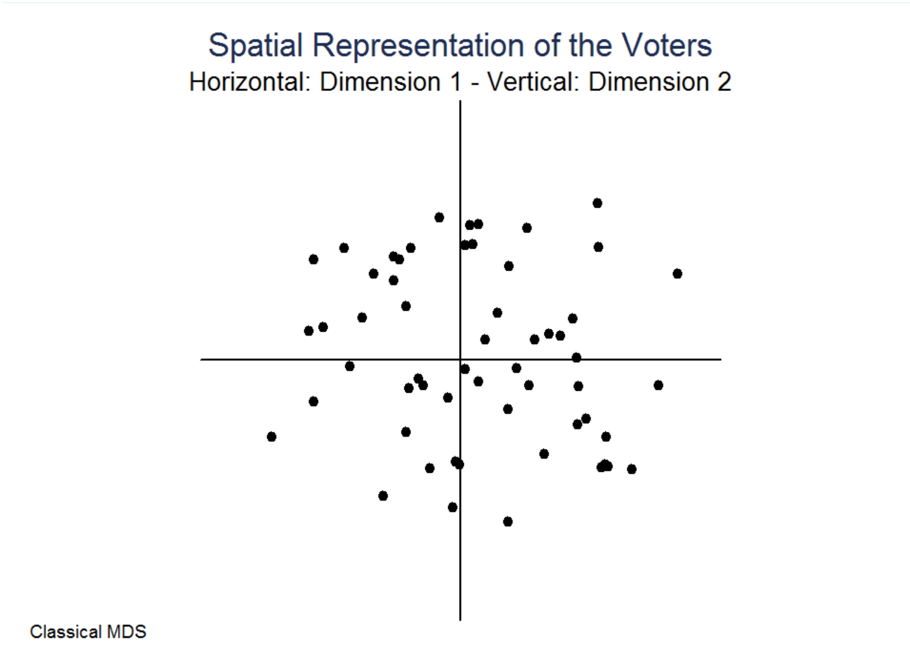

Figure 1: Voters: dimensions 1 - 2

Apart from the interpretation of the first principal component, one can notice that G and E, two researchers working in the field of computational social choice, are very close when considering the first two dimensions. Though, this observation has to be qualified since the coordinates with respect to the third axis considerably differ in the spatial points representing the two scholars (see Figure 3).

The most striking result can be seen in Figure 4, which shows the second and third dimensions of the variance maximizing, or distance preserving, projection from the space $\mathbb{R}^{63}$ into a three-dimensional body. We can disentangle the subgroup consisting of 5 alternatives that are located in the lower left quadrant. These five candidates, namely C, I, H, B and J, are female whereas the remaining seven persons running in the election are male. This is only a secondary effect (it does not appear on the first axis, the one correlated with scores, only in the next plane), but still, it means that the gender of the candidates is a secondary, but real, aspect that determines the votes.

5 Conclusion

The six studied rules are not equivalent: they may end up in the election of different candidates. But there is still much agreement: all these methods concur

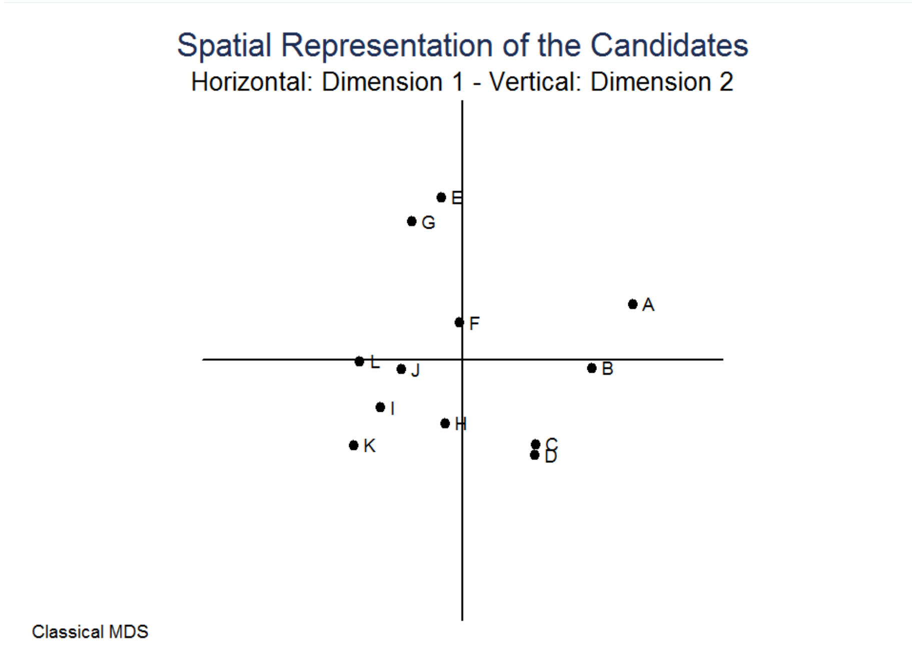

Figure 2: Candidates: dimensions 1 - 2

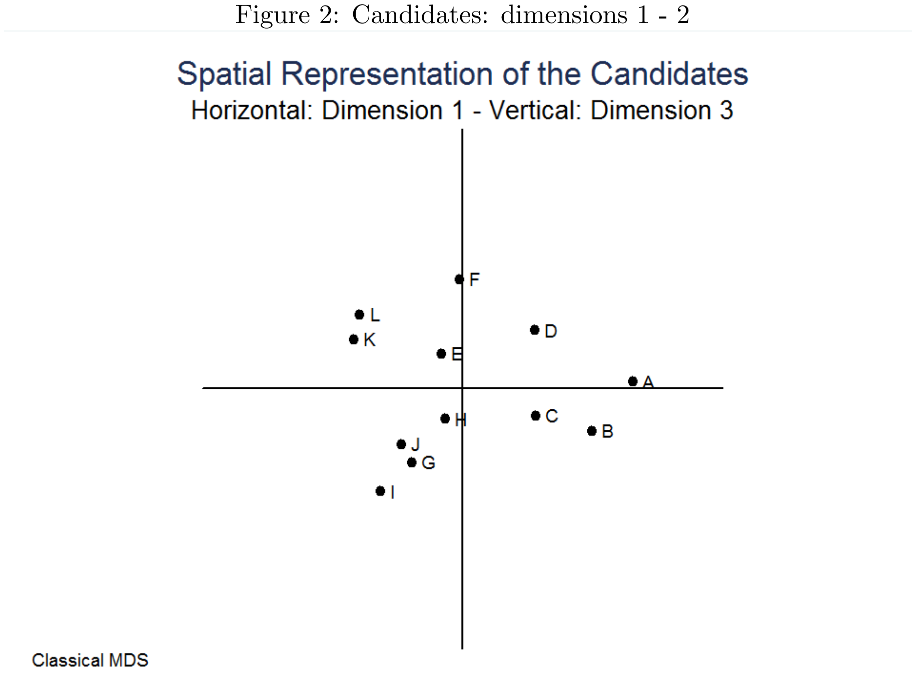

Figure 3: Candidates: dimensions 1 - 3

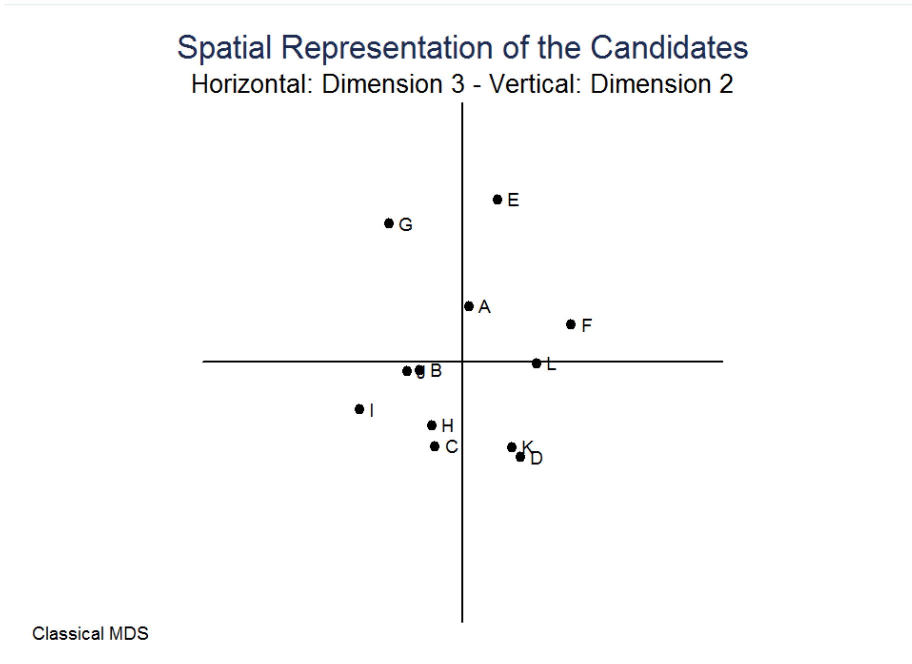

Figure 4: Candidates: Dimensions 3 - 2

in a reasonable way because their points of disagreement are on the candidates whose scores (or contributions, as we defined) are borderline between election and rejection. To confirm this observation we computed the six solutions on the data from 1999. There again, most candidates (8 out of 11) are elected either by all methods or by none of them. The methods differ about the fate of 3 candidates, and those are precisely the ones ranked seventh, eighth and ninth on the basis of the simple approval scores. The conclusion is thus that, in such "quiet" elections, the proposed methods all yield reasonable outcomes. Further studies, on different kinds of elections, should be made to understand what these methods do in practice.

The analysis of the vote profile reveals a rather homogeneous electorate but nevertheless unveils a gender effect in the correlations among candidates that is independent of the scores. This point confirms the interest of analyzing vote profiles when voters provide, with their ballots, more information than a single name.

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Classical metric multidimensional scaling
    dissimilarity: L1, computed on 12 variables

                                              Number of obs        =        63
    Eigenvalues > 0       =         12        Mardia fit measure 1 =    0.2911
    Retained dimensions   =          2        Mardia fit measure 2 =    0.6222
```

Dimension	Eigenvalue	abs(eigenvalue) Percent	Cumul.	(eigenvalue)^2 Percent	Cumul.
1	445.30181	17.94	17.94	44.86	44.86
2	277.04996	11.16	29.11	17.36	62.22
3	206.51012	8.32	37.43	9.65	71.87
4	170.58933	6.87	44.30	6.58	78.45
5	159.79999	6.44	50.74	5.78	84.23
6	130.98562	5.28	56.02	3.88	88.11
7	101.10255	4.07	60.09	2.31	90.42
8	76.586529	3.09	63.18	1.33	91.75
9	71.906684	2.90	66.07	1.17	92.92
10	63.513973	2.56	68.63	0.91	93.83

Figure 5: Multidimensional Scaling - Voters

```
Classical metric multidimensional scaling
    dissimilarity: L1, computed on 63 variables

                                              Number of obs        =        12
    Eigenvalues > 0       =          9        Mardia fit measure 1 =    0.6861
    Retained dimensions   =          3        Mardia fit measure 2 =    0.8777
```

Dimension	Eigenvalue	abs(eigenvalue) Percent	Cumul.	(eigenvalue)^2 Percent	Cumul.
1	1387.0678	28.91	28.91	43.76	43.76
2	1199.6026	25.01	53.92	32.73	76.48
3	704.55432	14.69	68.61	11.29	87.77
4	561.53113	11.71	80.31	7.17	94.94
5	355.37784	7.41	87.72	2.87	97.82
6	223.01466	4.65	92.37	1.13	98.95
7	109.7508	2.29	94.66	0.27	99.22
8	54.193323	1.13	95.79	0.07	99.29
9	1.2686794	0.03	95.81	0.00	99.29

Figure 6: Multidimensional Scaling - Candidates

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