

# CAN BEING IN PRISON ENTAIL MORE FREEDOM? <sup>①</sup>

- OPTIMAL BUNDLING WITH ANCHOR COSTS -

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## ABSTRACT\*

Based on my own experience, the question in the title admits an affirmative empirical answer. The paper proposes a model that explains why this can happen.

\* I wrote this paper while visiting the highly securitized department of the Silivri Prison. My visit was unintentional but the environment was inspiring about several human and social issues of different aspects. Moreover, my life conditions were very adequate to think, hence this paper. Having no access to the literature, formal citations are missing. Moreover, a better typesetting is needed. Nevertheless, I think this manuscript - the term being literally used - conveys my main point. Perhaps more important than its content, this document is a greeting to my dear colleagues some of whom form the closest friends of my life. You mean a lot to me in these moments of difficulty that, I hope, will not last long.

## 1. INTRODUCTION

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I have been arrested in October 2025 and spending my time in prison since then. What I am experiencing has very little legal grounds and I don't see much to report to a law review. On the other hand, during this time I made an observation about my own behavior, which I found worthy to analyze with the classical tools of economics. I hope this paper whose inspiration comes directly from real life and written in prison will be found to be of some interest by the decision theory community.

It has been years that I had the intention of reading back classical novels, which I couldn't do due to lack of time. Being in prison gave a compulsory break to many of my activities, which in turn allowed me to realize my dream. Very shortly, I remarked that replacing some of my activities with lectures of Herman Hesse made me happier. But in fact, I didn't need to be in prison to do this replacement. I always had the right to drop some of these activities, a right that I never exercised. Why?

Allocating our limited time among time consuming <sup>③</sup> activities is an application of the optimal bundling problem. As a general description, this problem consists of a decision maker confronting a list of options from which he is entitled to choose a bundle. There are constraints that render certain bundles infeasible and the decision maker aims to determine the feasible bundle that is best according to his own preference. For example, from a given list of candidates he may have to determine the best committee under the feasibility constraint imposed by the committee size. A generalization of this problem appears in participatory budgeting models where options are projects of different costs and the feasibility is determined by the total cost of a bundle that shouldn't exceed a given budget.\* The time allocation problem we consider is even more general, as the time consumed by a bundle of activities need not be additively calculated.

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\* The Ph.D. thesis of Matthieu Hervouin which, I heard, he brilliantly defended on 3 November 2015 has a chapter that contains relevant and recent references to this literature. Having to miss his defense will forever remain as a pain in my heart.

The optimal bundling problem endows the decision maker with a major difficulty. Even when he is able to compare any given pair of single options, extending this comparison the bundle pairs brings a considerable cognitive charge. The exponentially growing number of bundles to compare together with possible externalities arising from bundling makes the determination of the optimal bundle a complex task.\* This complexity increases in instances where the decision maker confronts the list of options to bundle not at once but over time in some undetermined order.\*\* In Grief, it is quite possible that the decision maker commits a miscalculation while determining an optimal bundle.

It is sometimes costless to revise the erroneous choice of a suboptimal bundle. For example, you can change your bundle of preferred projects submitted to the Paris municipality at any moment within the deadline and the only cost you incur is to press a few additional computer buttons. On the other hand, there are contexts where changing your choice can bring a higher cost such as

\* This complexity is the main motivation of the literature that adopts an axiomatic approach in handling this problem. Barberà, Bossert and Pattanaik have a relatively old but very comprehensive paper that surveys the literature on axiomatically extending an order over a set to its power set.

\*\* If I am not wrong, Saptarshi Mukherjee has a recent paper that addresses this question. We were planning his visit to LANSADE this year. I hope this will occur and I will also be there.

returning the goods you bought to the seller or ⑤  
the psychological Burden of resigning from a job.  
This is what we call the anchor cost of a bundle.\*

We analyze the optimal bundling problem under the existence of anchor costs. Our model is simple. From a given list of options we will choose a bundle. An exogenous feasibility constraint determines the bundles we are allowed to choose. The utility we get from a single option is known while the utility of a bundle can get any value within certain axiomatically defined limits. The anchor cost of leaving a chosen bundle is known.

An optimal bundle is one that gives us the highest utility. A bundle is stable if it doesn't give the incentive to switch to another one. For a bundle to be stable, either it must be optimal or the utility gain from switching shouldn't justify the anchor cost induced by this switch. We aim to pick bundles that are optimal and, if not, at least we wish that our choice is unstable. In other words, we don't want to get stuck at suboptimality by choosing a bundle that is not optimal but stable. Thus, we qualify as acceptable bundles that are optimal or unstable.

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\* One should be careful for not falling in the fallacy of including sunk costs to the anchor cost.

⑥ We consider decision rules that choose a bundle as a function of the feasibility constraint, the utilities of the options and the anchor costs of the bundles. We ask the chosen bundle to be acceptable for every possible utility of the bundles coming from an axiomatically determined interval. We also ask the chosen bundle to be option-maximizing, i.e., not to admit a feasible superset. Our main result expressed by Theorem 1 is negative: With at least three options, there exist no decision rule that ensures to choose of acceptable and option-maximizing bundles.

Expanding the limits of the interval from which bundles get their utility values can only make our task harder. On the other hand, narrowing these limits could be of help. Theorem 2 brings a reserved optimism in this direction: If the preference over options determines the strict part of the preference over bundles then there exists a unique decision rule that ensures the choice of an acceptable and option-maximizing bundle. The positive result relies on the fact that the preference over bundles is almost known. In fact, the impossibility is back in case the preference over options allows two bundles to be ranked oppositely.

Finally, we consider the impact of being content with acceptability without being option-maximizing. Theorem 3 announces that a decision rule that ensures acceptability has to deliver unstable bundles. However, decision problems with high anchor costs may not admit an unstable bundle. Thus, as expressed by Theorem 4, the existence of a decision rule that ensures acceptable bundles means a domain consisting of decision problems with low anchor costs.

Section 2 introduces the model. Section 3 defines optimality and stability, establishing their logical relationship. Section 4 considers decision rules and contains our main results. Section 5 makes some concluding remarks.

## 2. THE MODEL

(8)

For an integer  $m \geq 2$ , we let  $A = \{a_1, \dots, a_m\}$  be the set of mutually inclusive options that an individual can potentially choose. The choice of multiple options is allowed subject to a given feasibility constraint expressed by a non-empty set  $B \subseteq 2^A$  of feasible bundles, satisfying

$$X \in B \Rightarrow Y \in B \quad \forall X, Y \in 2^A \text{ with } Y \subseteq X.$$

The individual has a linear preference over  $A$  expressed by  $u: A \rightarrow \mathbb{R}_{++}$ . A preference  $u$  is extended over bundles by  $U: 2^A \setminus \{\emptyset\} \rightarrow \mathbb{R}_{++}$  that satisfies

$$(i) \quad U(\{x\}) = u(x) \quad \forall x \in A \quad (\text{consistency})$$

$$(ii) \quad X \supseteq Y \Rightarrow U(X) \geq U(Y) \quad \forall X, Y \in 2^A \setminus \{\emptyset\} \quad (\text{option monotonicity})$$

$$(iii) \quad U(X) \leq \sum_{x \in X} u(x) \quad \forall X \in 2^A \setminus \{\emptyset\} \quad (\text{no positive externality})$$

Note that while (iii) imposes an upper bound on the value of  $U(X)$ , (i) and (ii) imply  $\max_{x \in X} \{u(x)\} \leq U(X)$   $\forall X \in 2^A \setminus \{\emptyset\}$  as a lower bound.

For any  $u$ , we write  $\Pi(u) = \{U: 2^A \setminus \{\emptyset\} \rightarrow \mathbb{R}_{++}: U \text{ satisfies } \textcircled{9} \}$   
 (i), (ii) and (iii)

for the set of possible preference extensions over  $2^A \setminus \{\emptyset\}$ .

The individual can discard a bundle  $X \in 2^A \setminus \{\emptyset\}$  that he had previously chosen subject to an anchor cost expressed by  $C: 2^A \setminus \{\emptyset\} \rightarrow \mathbb{R}_+$  satisfying  $C(X) \geq C(Y) \forall X, Y \in 2^A \setminus \{\emptyset\}$  with  $X \supseteq Y$ .

### 3. OPTIMALITY AND STABILITY

(10)

Take some  $B$ . Given some  $U: 2^A \setminus \{\emptyset\} \rightarrow \mathbb{R}_{++}$ , a

bundle  $X \in B$  is  $U$ -optimal iff  $U(X) \geq U(Y)$

$\forall Y \in B$ . So there may be multiple  $U$ -optimal bundles.

Given a pair  $(U, C)$  of a preference over bundles and anchor cost function of bundles, a bundle

$X \in B$  is  $(U, C)$ -stable iff  $U(X) \geq U(Y) - C(X \setminus Y)$

$\forall Y \in B$ . Switching the choice from  $X$  to  $Y$  requires to discard the bundle  $X \setminus Y$ , imposing the anchor cost  $C(X \setminus Y)$ . In case  $C(X \setminus Y)$  is no less than the gain  $U(Y) - U(X)$  of switching, the individual has no incentive to switch, hence the stability condition.

Proposition 1: For any  $(U, C)$ ,

- (i)  $U$ -optimality implies  $(U, C)$ -stability under any  $B$ .
- (ii)  $[(U, C)$ -stability implies  $U$ -optimality under any  $B] \Leftrightarrow C(X) = 0 \quad \forall X \in 2^A \setminus \{\emptyset\}$ .

Proof: (i) follows from the fact that anchor costs <sup>①</sup> are non-negative. For (ii), ( $\Leftarrow$ ) is obvious. To see ( $\Rightarrow$ ), let  $B = \{\emptyset, \{a_1\}, \{a_2\}\}$  for some  $a_1, a_2 \in A$  with  $C(\{a_2\}) > 0$ . Let  $U(\{a_1\}) = 1 + \frac{C(\{a_2\})}{2}$  and  $U(\{a_2\}) = 1$ , rendering  $\{a_2\}$   $(U, C)$ -stable while  $\{a_1\}$  is the only  $U$ -optimal bundle. Q.E.D.

So stability and optimality are equivalent if and only if there are no anchor costs. Otherwise, optimality is stronger.

A stable but non-optimal bundle is an instance where "being in prison can entail more freedom" by exogenously restricting  $B$  so that this non-optimal stable bundle vanishes. When  $B$  in the proof of Proposition 1 is restricted to  $\{\emptyset, \{a_1\}\}$ , the outcome will have to switch from  $\{a_2\}$  to  $\{a_1\}$ , increasing the welfare of the individual.

We wish to rule out non-optimal stable outcomes. Given  $(U, C)$ , we qualify a bundle  $X \in B$  as  $(U, C)$ -acceptable iff  $X$  is  $U$ -optimal or  $(U, C)$ -unstable.

#### 4. CHOOSING A BUNDLE: DECISION RULES (12)

Given  $u: X \rightarrow \mathbb{R}_+$  and  $C: 2^X \setminus \{\emptyset\} \rightarrow \mathbb{R}_+$ , we call a pair  $(u, C)$  a problem. A problem  $(u, C)$  is non-trivial iff  $C(X) > 0 \ \forall X \in 2^X \setminus \{\emptyset\}$ .

A domain is a non-empty set  $D$  of non-trivial problems.

A problem  $(u, C)$  is stability allowing iff every  $X \in 2^X \setminus \{\emptyset\}$  is  $(u, C)$ -stable for some  $U \in \Pi(u)$ . Being stability allowing means that  $C$  reaches values that are sufficiently high than those of  $u$ . A domain  $D$  is stability allowing iff it consists of stability allowing problems.

Fix some  $D$ . For each  $B$ , define a mapping  $\delta_B: D \rightarrow B$  that assigns to each problem  $(u, C) \in D$  a feasible bundle  $\delta_B(u, C) \in B$ .

A decision rule is a family  $\delta = \{\delta_B\}$  defined for each  $B$ . We qualify  $\delta$  as acceptable iff given any  $B$ ,  $\delta_B(u, C)$  is  $(u, C)$ -acceptable,  $\forall (u, C) \in D, \forall U \in \Pi(u)$ .

(13)

The following result is about the structure of acceptable decision rules.

Proposition 2: Take any  $D$ . A decision function  $\delta$  is acceptable  $\Leftrightarrow$  given any  $B$  and  $\forall (u, C) \in D$ ,  $\delta_B(u, C)$  is either  $U$ -optimal  $\forall U \in \Pi(u)$  or  $(U, C)$ -unstable  $\forall U \in \Pi(u)$ .

Proof: The "if" part is left to the reader. To see the "only if" part take any  $B$ , any  $(\bar{u}, C) \in D$  and suppose  $\delta_B(\bar{u}, C)$  is  $U$ -optimal for some  $U \in \Pi(\bar{u})$  and  $(U, C)$ -unstable for some  $U \in \Pi(\bar{u})$ . Writing  $\delta_B(\bar{u}, C) = X$ , this implies the existence of  $Y \in B$  with  $U(Y) - U(X) > C(X|Y)$  while  $U(X) \geq U(Y)$ . In case of multiplicity, pick  $Y$  with highest  $U(Y)$ .

Case 1:  $\sum_{x \in X} \bar{u}(x) < U(Y)$ . Set  $W(X) = \sum_{x \in X} \bar{u}(x)$ . As

$U(X) \geq U(Y)$ , we have  $\sum_{x \in X} \bar{u}(x) \geq \max_{y \in Y} \bar{u}(y)$ . Thus, we

can set  $W(Y) = \lambda U(Y) + (1-\lambda) \sum_{x \in X} \bar{u}(x)$  for some  $\lambda \in [0, 1]$ .

So  $W(Y) - W(X) = \lambda (U(Y) - \sum_{x \in X} \bar{u}(x)) \leq C(X|Y)$  for

$\lambda$  sufficiently small. Set  $W(z) = U(z) \forall z \in 2^X \setminus \{X, Y\}$ .  $X$  is  $(W, C)$ -stable but not  $W$ -optimal for  $W \in \Pi(\bar{u})$ .

Case 2:  $\sum_{x \in X} u(x) \geq U(Y)$ . Set  $W(Y) = U(Y)$  and  $\textcircled{15}$

$W(X) = \lambda U(X) + (1-\lambda) U(Y)$  for some  $\lambda \in [0, 1]$ .

Note that  $W(Y) - W(X) = \lambda (U(Y) - U(X)) \leq C(X|Y)$

when  $\lambda$  is sufficiently small. Also set  $W(Z) = U(Z)$

$\forall Z \in 2^A \setminus \{X, Y\}$ . So  $W(Y) \geq W(Z)$  for every  $Z \in 2^A$

with  $U(Z) - U(X) > C(X|Z)$ . Thus  $X$  is  $(W, C)$ -stable

but not  $W$ -optimal for  $W \in \Pi(u)$ . Q.E.D.

Remark: When  $D$  is stability allowing, acceptability of  $\delta$  implies for any  $B$  that  $\delta_B(u, C)$  is  $U$ -optimal  $\forall U \in \Pi(u)$ . Thus, Proposition 2 holds trivially for stability allowing  $D$ .

$\delta$  is option-maximizing iff given any  $B$ ,  $\delta_B(u, C) \subset X$  fails  $\forall X \in B$ ,  $\forall (u, C) \in D$ . Note that under option-monotonicity of preferences over bundles, the condition weakens the requirement of choosing only optimal bundles. This condition, combined with acceptability, leads to an impossibility.

Theorem 1: Let  $\#A \geq 3$ . Take any  $D$ . There exist no  $\delta$  that is option-maximizing and acceptable.

(15)

Proof: Take any  $D$ , any  $(u, C) \in D$  and any  $a_1, a_2, a_3 \in A$  with  $u(a_1) > u(a_2) > u(a_3)$ . Let  $B = \{\emptyset, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_1, a_3\}\}$ . As  $S$  is option-maximizing,  $S_B(u, C) \in \{\{a_1, a_2\}, \{a_1, a_3\}\}$ .

First let  $S_B(u, C) = \{a_1, a_2\}$ . Set  $U(\{a_1, a_2\}) = u(a_1)$ .

If  $u(a_3) < C(\{a_2\})$  then set  $U(\{a_1, a_3\}) = u(a_1) + u(a_3)$  and note that  $U(\{a_1, a_3\}) - U(\{a_1, a_2\}) = u(a_3) < C(\{a_2\})$ , rendering  $\{a_1, a_2\}$   $U$ -nonoptimal but  $(U, C)$ -stable while  $U \in \Pi(u)$ .

If  $u(a_3) \geq C(\{a_2\})$  then set  $U(\{a_1, a_3\}) = u(a_1) + \frac{C(\{a_2\})}{2}$ . Note that  $U(\{a_1, a_3\}) -$

$U(\{a_1, a_2\}) = \frac{C(\{a_2\})}{2} < C(\{a_2\})$ , again rendering

$\{a_1, a_2\}$   $U$ -nonoptimal but  $(U, C)$ -stable while  $U \in \Pi(u)$ .

Now let  $S_B(u, C) = \{a_1, a_3\}$ . Set  $U(\{a_1, a_3\}) = u(a_1)$ .

If  $u(a_2) < C(\{a_3\})$  then set  $U(\{a_1, a_2\}) = u(a_1) + u(a_2)$ , ensuring  $U(\{a_1, a_2\}) - U(\{a_1, a_3\}) = u(a_2) < C(\{a_3\})$ .

If  $u(a_2) \geq C(\{a_3\})$  then set  $U(\{a_1, a_2\}) = u(a_1) + \frac{C(\{a_3\})}{2}$ ,

ensuring  $U(\{a_1, a_2\}) - U(\{a_1, a_3\}) = \frac{C(\{a_3\})}{2} < C(\{a_3\})$  while  $U \in \Pi(u)$ . Q.E.D.

Remark: For  $\#A=2$ , there exists a unique  $\mathcal{S}$  that is option-maximizing and acceptable. We leave its construction to the reader.

Given any  $u$ , the larger is the set of possible preference extensions over bundles the stronger is acceptability. As a result, the impossibility advanced by Theorem 1 prevails when  $\Pi$  is enlarged, for example by dispensing with the no positive externality condition. On the other hand, narrowing  $\Pi$  can lead to a positive result, a direction that we take in the sequel.

Consider a mapping  $\mathcal{S}$  from the set of linear orders over  $A$  to the set of linear orders over  $2^A \setminus \{\emptyset\}$ . A complete preorder  $R$  over  $2^A \setminus \{\emptyset\}$  is  $\mathcal{S}$ -compatible with a linear order  $>$  over  $A$  iff  $R$  doesn't reverse any strict pairwise comparison made by  $\mathcal{S}(>)$ . We write  $\succ_u$  for the linear order over  $A$  induced by  $u$  and  $R_u$  for the complete preorder over  $2^A \setminus \{\emptyset\}$  induced by  $u$ . For each  $u$ , we let  $K_{\mathcal{S}}(u) = \{u \in \Pi(u) : R_u \text{ is } \mathcal{S}\text{-compatible with } \succ_u\}$ . We ensure the non-emptiness of  $K_{\mathcal{S}}(u)$  by imposing option-monotonicity over  $\mathcal{S}$ , i.e.,  $X \mathcal{S}(\succ_u) Y \implies \forall X' Y' \in 2^A \setminus \{\emptyset\} \text{ with } X' \supset Y'.$

Theorem 2: Take any  $K_S$  and any  $D$ . There exists a unique option-maximizing and acceptable  $S$ .

Moreover, for  $K \supset K_S$ , there exist no option-maximizing and acceptable  $S$ .

Proof: Take any  $K_S$  and any  $D$ . For any  $(u, C) \in D$  and any  $B$ , set  $S_B(u, C) = X = \operatorname{argmax}_{Z \in B} S(Z_u)$ .

So  $U(X) \geq U(Y) \forall Y \in B \forall U \in K_S(u)$ , ensuring the acceptability of  $S$ . The option-monotonicity of  $S$  ensures that  $S$  is option-maximizing. To see the uniqueness, suppose  $S_B(u, C) = Y \neq X$ . Let  $\epsilon = \min_{Z \in B} \{C(Y \setminus Z)\}$ . As  $X S(Z_u) Y$ ,  $U(X) \geq U(Y)$

$\forall U \in K_S(u)$ . Pick some  $V \in K_S(u)$  with  $0 < U(X) - U(Y) \leq \epsilon \leq C(Y \setminus X)$ . As  $U(X) \geq U(Z) \forall Z \in B$ ,  $U(Z) - U(Y) \leq \epsilon \leq C(Y \setminus Z) \forall Z \in B$  as well. Thus,  $Y$  is  $(U, C)$  stable but not  $U$ -optimal, stable but not  $U$ -optimal.

Now take any  $K \supset K_S$ , any  $(\bar{u}, C) \in D$  and any  $U \in K(\bar{u}) \setminus K_S(\bar{u})$ . As  $U \notin K_S(\bar{u})$ , pick some  $X, Y \in \bigcup_{Z \in B} S(Z_{\bar{u}})$  with  $X S(Z_{\bar{u}}) Y$  while  $U(Y) > U(X)$ .

As  $X \succ(\succ_u) Y$ ,  $\exists U \in K_P(\bar{u})$  with  $U(X) \overset{(18)}{>} U(Y)$ .

Note that  $\#X = \#Y = 1$  cannot hold. Now let

$B = \{Z \in 2^A : Z \subseteq X \text{ or } Z \subseteq Y\}$ . As  $\delta$  is option-maximizing,  $\delta_B(\bar{u}, C) \in \{X, Y\}$ . Let  $\delta_B(\bar{u}, C) = X$ . Having  $U(X) > U(Y)$  ensures  $\max_{y \in Y} \{u(y)\} < \sum_{x \in X} u(x)$ .

Thus  $\exists W \in K(\bar{u})$  with  $W(Y) - W(X) \leq C(X \setminus Y)$ .

Now let  $\delta_B(\bar{u}, C) = Y$ . Having  $U(Y) > U(X)$  ensures  $\max_{x \in X} \{u(x)\} < \sum_{y \in Y} u(y)$ . Thus,  $\exists W \in K(\bar{u})$

with  $W(X) - W(Y) \leq C(Y \setminus X)$  Q.E.D.

So the only restriction of  $\pi$  that enables to avoid the impossibility enounced by Theorem 1 is when we can deduce from  $u$  the strict counterpart of the ranking over  $2^A \setminus \{\emptyset\}$ .

A particular  $\delta$  of interest is the lexicographic one. Instead of giving its tedious definition, we illustrate it through the following example with  $\#A = 3$ .

(19)

| $\gamma$ | $\lambda(\gamma)$   |
|----------|---------------------|
| $a_1$    | $\{a_1, a_2, a_3\}$ |
| $a_2$    | $\{a_1, a_2\}$      |
| $a_3$    | $\{a_1, a_3\}$      |
|          | $\{a_1\}$           |
|          | $\{a_2, a_3\}$      |
|          | $\{a_2\}$           |
|          | $\{a_3\}$           |

When  $\Pi$  is restricted by being replaced with  $K_1$ , the outcome of the unique option-maximizing and acceptable  $\mathcal{S}$  can be obtained as follows:  
 Given  $u$ , consider  $\gamma_u$  and start picking the options from the top down to the bottom subject to  $B$ . Stop when you reach the very end or  $B$  doesn't allow you to pick any further options.

We now consider the impact of dispensing with  $\mathcal{S}$  being option-maximizing.

Theorem 3: Take any  $D$ .  $\mathcal{S}$  is acceptable  $\Leftrightarrow$  given any  $B$ ,  $\mathcal{S}_B(u, C)$  is  $(u, C)$ -unstable  $\forall u \in \Pi(u), \forall (u, C) \in D$ .

Proof: We leave the "if" part to the reader. To see the "only if" part take any  $D$ , any  $(u, C) \in D$  and any  $B$ . Let  $S$  be acceptable. Write  $D_1 = \{(u, C) \in D : S_B(u, C) \text{ is } u\text{-optimal } \forall U \in \Pi(u)\}$  and  $D_2 = \{(u, C) \in D : S_B(u, C) \text{ is } (u, C)\text{-unstable } \forall U \in \Pi(u)\}$ . By Proposition 2,  $D_1 \cup D_2 = D$ . We complete the proof by showing  $D_1 = \emptyset$ . Suppose not and define  $\bar{S}_B : D_1 \rightarrow B$  as  $\bar{S}_B(u, C) = S_B(u, C) \forall (u, C) \in D_1$ . As  $S$  is acceptable,  $\bar{S} = \{\bar{S}_B\}$  is also acceptable. By construction of  $D_1$ ,  $\bar{S}_B(u, C)$  is  $u$ -optimal  $\forall U \in \Pi(u)$ ,  $\forall (u, C) \in D_1$  implying  $\bar{S}$  to be option-maximizing, contradicting Theorem 1. Q.E.D.

Theorem 4: Take any  $D$ . There exists an acceptable  $S$   $\iff$   $D$  consists of problems that fail to be stability allowing.

Proof: Take any  $D$ . To see the "only if" part, let  $(u, C) \in D$  be stability allowing and  $S$  be acceptable. By Theorem 3, for any  $B$ ,  $S_B(u, C)$  is  $(u, C)$ -unstable  $\forall U \in \Pi(u)$ , contradicting that  $(u, C)$  is stability allowing. We now show the "if" part. As every  $(u, C)$  fails to be stability allowing,  $\exists X(u, C) \in B$  that is  $(u, C)$ -unstable  $\forall U \in \Pi(u)$ . Set  $S_B(u, C) = X(u, C)$ , rendering  $S$  acceptable by Theorem 3. Q.E.D.

## 5. CONCLUDING REMARKS

(2)

Our findings reflect a negative result in decision theory: When it is costly to discard chosen options, it is not possible to guarantee the choice of a bundle that is optimal or at least unstable. In other words, one risks to get stuck with a suboptimal bundle, a fact that we experience in our daily lives.

The impossibility stems from the limited information available to the decision rule: the bundle is chosen as a function of the preference over options while optimality and stability are determined with respect to the preference over bundles. As a matter of fact, Theorem 1 is another negative reflection of the ambiguity on extending an order over a set to its power set.\*

As expected, the impossibility is avoided when this ambiguity is minimized so that the ranking of options determines the strict part of the ranking of bundles. This means almost knowing how bundles are ranked in which case the ability of making an optimal choice is unsurprising. (Theorem 2).

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\* In a recent paper, Jean Laroche, Jérôme Lang, İpek Ötkel Şener and I consider a collective choice model to consider the possibility of choosing a set that is Pareto optimal with respect to individuals' preferences over sets based on their preferences over alternatives.

Another way to avoid the impossibility is to be <sup>(2)</sup> content with instability without optimality. However, as Theorem 3 and Theorem 4 indicate not every problem admits unstable bundles and even when it does, aiming to choose a bad bundle with low anchor cost is barely satisfactory.